The equations for determining the density from an interferogram obtained with this type of interferometer are the same as those obtained for the Mach-Zehnder interferometer. For the infinite fringe setting, the fringes resulting from a two-dimensional disturbance represent lines of constant density, the values of which may be obtained from the following general two-dimensional equation[3]:

$$\frac{\rho_1}{\rho_\infty} = \frac{(S_1 - S_\infty)\lambda}{L(n-1)} + 1 \tag{4}$$

If the components of the interferometer are first arranged and adjusted similar to the usual schlieren system, it is only necessary to insert the light stops and grating to obtain fringes. In general, the only other adjustments needed are a translation motion of the grating to obtain the desired fringe spacing and the focusing of the disturbance on the screen.

Figure 3a shows a typical interferogram of a free jet exhausting against a flat plate taken with this instrument. The interferogram was taken with the interferometer adjusted for the infinite fringe pattern. Some of the characteristics of the jet are also sketched in Fig. 3b. A commercial replica transmission grating with 2000 lines per inch was used in the interferometer.

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Deflection of a Uniformly Loaded Rectangular Plate with One Pair of Parallel Edges Rigidly Connected to Two Identical Beams

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Rao discussed the bending of rectangular plates with edge beams but did not give any specific solution.[1] Haskin considered the nonlinear problem of rectangular plates stiffened by beams along two edges,[2] and his work is extensive. However, it is believed that the simple linear solution is still technically valuable. This note presents a series solution for a uniformly loaded rectangular plate with two parallel edges built into two identical, elastic beams of constant cross section and simply supported at the other two edges. This solution embraces the solutions for the following four special cases, namely, 1) uniformly loaded rectangular plate with two parallel edges supported on two identical, elastic beams and the other two simply supported, 2) uniformly loaded rectangular plate with two parallel edges free and the other two simply supported, 3) uniformly loaded rectangular plate simply supported at all of the edges, and 4) uniformly loaded rectangular plate with two parallel edges clamped and the other two simply supported. The solution is obtained as follows: The differential equation for small deflections of a thin plate is

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D} \tag{1}$$

The required boundary conditions for the problem under consideration are

$$w = 0 \qquad \partial^2 w/\partial x^2 = 0 \quad \text{at } x = 0 \text{ and } a \tag{2}$$

$$EI\left(\frac{\partial^4 w}{\partial x^4}\right)_{y=\pm b/2} = D\left[\frac{\partial^3 w}{\partial y^3} + (2-\nu)\frac{\partial^3 w}{\partial x^2 \partial y}\right]_{y=\pm b/2} \tag{3}$$

$$-C\frac{\partial}{\partial x}\left(\frac{\partial^2 w}{\partial x \partial y}\right)_{y=\pm b/2} = D\left[\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2}\right]_{y=\pm b/2} \tag{4}$$

Equation (2) describes the simply supported condition at $x = 0$ and $x = a$. Equations (3) and (4) describe the rigid attachment of the two edges $y = \pm b/2$ to the elastic beams. In the preceding equations, w is the deflection of the plate; q is the intensity of the uniform load; EI and C are the flexural and torsional rigidity of the plate, respectively; and ν is Poisson's ratio. The dimensions of the plate are $a \times b$. Assume a solution in the form

$$w = \frac{qa^4}{D}\sum_{m=1,3,5\ldots}^{\infty}\left(\frac{4}{m^5\pi^5} + A_m\cosh\frac{m\pi y}{a} + B_m\frac{m\pi y}{a} \times \sinh\frac{m\pi y}{a}\right)\sin\frac{m\pi x}{a} \tag{5}$$

where A_m and B_m are arbitrary constants. It can be verified that the assumed solution satisfies the differential equation and the boundary conditions at $x = 0$ and $x = a$.

Substitution of w from Eq. (5) into Eqs. (3) and (4) results in the following two equations:

$$A_m[(\nu-1)\sinh\alpha_m - m\pi\lambda\cosh\alpha_m] + B_m\{\alpha_m(\nu-1)\cosh\alpha_m + [(\nu+1) - m\pi\lambda\alpha_m]\sinh\alpha_m\} = 4\lambda/m^4\pi^4 \tag{6}$$

$$A_m[m\pi\gamma\sinh\alpha_m + (\nu-1)\cosh\alpha_m] + B_m[(m\pi\gamma\alpha_m - 2)\cosh\alpha_m + m\pi\gamma - \alpha_m(1-\nu)\sinh\alpha_m] = -4\nu/m^4\pi^5 \tag{7}$$

where

$$\lambda = EI/Da \qquad \gamma = C/Da \qquad \alpha_m = m\pi b/2a$$

From Eqs. (6) and (7) a simultaneous solution yields the arbitrary constants A_m and B_m as

$$A_m = (4/m^5\pi^5\Delta)\{[m^2\pi^2\lambda\gamma\alpha_m - 2m\pi\lambda - \alpha_m\nu(1-\nu)]\cosh\alpha_m - [m\pi\lambda\alpha_m - m^2\pi^2\lambda\gamma - \nu(1+\nu)]\sinh\alpha_m\} \tag{8}$$

$$B_m = (4/m^5\pi^5\Delta)\{[-m^2\pi^2\lambda\gamma + \nu(1-\nu)]\sinh\alpha_m + m\pi\lambda\cosh\alpha_m\} \tag{9}$$

where

$$\Delta = [(1-\nu)(\nu+3) - m^2\pi^2\lambda\gamma]\cosh\alpha_m\sinh\alpha_m - \alpha_m(1-\nu)^2 - m^2\pi^2\lambda\gamma\alpha_m - 2m\pi\gamma\sinh^2\alpha_m + 2m\pi\lambda\cosh^2\alpha_m \tag{10}$$

Substituting A_m and B_m into Eq. (5), the desired solution of the problem under consideration is obtained as

$$w = \frac{4qa^4}{D\pi^5}\sum_{m=1,3,5\ldots}^{\infty}\frac{1}{m^5}\left[1 + \frac{1}{\Delta}\left(\{[m^2\pi^2\lambda\gamma\alpha_m - 2m\pi\lambda - \alpha_m\nu(1-\nu)]\cosh\alpha_m - [m\pi\lambda\alpha_m - m^2\pi^2\lambda\gamma - \nu(1+\nu)]\sinh\alpha_m\}\cosh\frac{m\pi y}{a} + \{[-m^2\pi^2\lambda\gamma + \nu(1-\nu)] \times \sinh\alpha_m + m\pi\lambda\cosh\alpha_m\}\frac{m\pi y}{a}\sinh\frac{m\pi y}{a}\right)\right]\sin\frac{m\pi x}{a} \tag{11}$$

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Setting $\gamma = 0$, Eq. (11) yields the solution to special case (1).[3] Setting both $\gamma = 0$ and $\lambda = 0$, Eq. (11) yields the solution to special case (2).[4] Setting $\gamma = 0$ and letting $\lambda \to \infty$, Eq. (11) yields the solution to special case (3).[5] Finally, letting both $\gamma \to \infty$ and $\lambda \to \infty$, Eq. (11) yields the solution to special case (4). In this last case the resulting expression for the deflection becomes

$$w = \frac{4qa^4}{D\pi^5} \sum_{m=1,3,5}^{\infty} \frac{1}{m^5} \left\{ 1 - \left[\frac{(\alpha_m \cosh\alpha_m + \sinh\alpha_m) \cosh(m\pi y/a) - \sinh\alpha_m (m\pi y/a) \sinh(m\pi y/a)}{\cosh\alpha_m \sinh\alpha_m + \alpha_m} \right] \right\} \sin\frac{m\pi x}{a} \quad (12)$$

which is simpler than that which is offered in Timoshenko's text[6] where the total deflection is given as a superposition of two separate solutions. However, after a certain amount of algebraical work, it has been shown that Timoshenko's result could be simplified to agree with Eq. (12).

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Prediction of Adiabatic Wall Temperatures in Film-Cooling Systems

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Nomenclature

u_C = velocity of coolant fluid emerging from slot, presumed uniform across slot
u_G = velocity of mainstream, presumed uniform along plate
x = distance along plate measured from slot
X = dimensionless distance defined by Eq. (9)
y_C = width of slot in direction normal to wall
ϵ = effectiveness of film cooling, i.e., temperature difference between the adiabatic wall and mainstream, divided by temperature difference between coolant in the slot and mainstream
ν = kinematic viscosity of fluid, presumed uniform

1. Introduction

In a recent paper in this journal, Librizzi and Cresci[1] described a simple theory, based on a suggestion by Libby, for the calculation of the adiabatic wall temperature produced by film cooling. The theory starts from the assumption that the coolant fluid is fully mixed with the material in the boundary layer, which, therefore, exhibits a uniform temperature along a normal to the wall; a discontinuity of temperature must appear at the outer "edge" of the boundary layer. The growth of the boundary layer is supposed to follow the usual laws, without any special influence of injection. The authors successfully compared the predictions of the theory with data published by Nishiwaki et al.[2]

The purpose of the present note is threefold: 1) to mention some independent developments of the same theory, 2) to report on the successes and failures achieved with it, and 3) to mention some later methods of predicting the thermal effects of film cooling. The writer believes that the conceptual simplicity of the theory of Ref. 1 renders it a valuable starting point for understanding film cooling; however, its limitations have to be recognized.

In order to emphasize the similarities in the theories, discussion is here restricted to two-dimensional uniform-property flows.

2. Earlier Theories

Stollery[3] and Kutateladze and Leont'ev[4] have developed theories that are almost identical in starting points and results with those of Ref. 1. The similarities may be seen by expressing the results of each author in the form of an equation for the film-cooling effectiveness ϵ in terms of the Reynolds number of the coolant fluid at the slot exit $u_C y_C/\nu$, the ratio of coolant velocity to mainstream velocity u_C/u_G, and the ratio of downstream distance to slot width x/y_C.

Stollery deduced

$$\epsilon = 3.09 \left(\frac{u_G}{u_C}\frac{x}{y_C}\right)^{-0.8} \left(\frac{u_C y_C}{\nu}\right)^{0.2} \quad (1)$$

Kutateladze and Leont'ev deduced

$$\epsilon = 3.1 \left\{4.16 + \frac{u_G}{u_C}\frac{x}{y_G} \left(\frac{u_C y_C}{\nu}\right)^{-0.25}\right\}^{-0.8} \quad (2)$$

Librizzi and Cresci deduced

$$\epsilon = 3.0 \left\{3.0 + \left(\frac{u_G}{u_C}\frac{x}{y_C}\right)^{0.8} \left(\frac{u_C y_C}{\nu}\right)^{-0.2}\right\}^{-1} \quad (3)$$

It is easily seen that, when x/y_C is large, the three expressions lead to almost identical expressions for ϵ. The differences derive from slightly divergent treatments of the upstream region and from trivial differences in the multiplying constant. Incidentally, Stollery has pointed out in another report[5] that similar formulas are implicit in the works of earlier authors, namely, those of the following:

Wieghardt[6] (together with the assumption that the boundary-layer thickness equals $0.37x(ux/\nu)^{-1.2}$)

$$\epsilon = 5.44 \left(\frac{u_G}{u_C}\frac{x}{y_C}\right)^{-0.8} \left(\frac{u_C y_C}{\nu}\right)^{0.2} \quad (4)$$

Hartnett et al.[7]

$$\epsilon = 3.39 \left(\frac{u_G}{u_C}\frac{x}{y_C}\right)^{-0.8} \left(\frac{u_C y_C}{\nu}\right)^{0.2} \quad (5)$$

Tribus and Klein[8]

$$\epsilon = 4.62 \left(\frac{u_G}{u_C}\frac{x}{y_C}\right)^{-0.8} \left(\frac{u_C y_C}{\nu}\right)^{0.2} \quad (6)$$

The differences in the coefficient derive from different assumptions about the temperature profile, i.e., about the extent to which the coolant fluid is fully mixed with the fluid in the boundary layer. The better the mixing, the lower the coefficient; this is why the theory of Libby-Stollery-Kutateladze, which assumes complete mixing, gives the lowest coefficient of all.

3. Comparison with Experiment

Kutateladze and Leont'ev compared their formula with the same data as were selected by Librizzi and Cresci, namely

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